Monte Carlo Simulation

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BACKGROUND

In general terms, the Monte Carlo method (or Monte Carlo simulation) is any problem-solving technique that approximates the solution through statistical sampling. That is, the method translates uncertain inputs (defined by probability distributions) into an output (also described by a probability distribution).

This means that outcomes previously expressible only in qualitative terms ("if we do X then Y might result") can be put as quantified probabilities ("if we do X there is a 20% chance that Y will result"). Typically, this proves far more useful to decision-makers.

HISTORY

Monte Carlo simulation is named after the city, famous for its casino. Although there are earlier examples of similar principles having been engaged, the modern instantiation of the method dates from the Manhattan Project, when the mathematician Stanislaw Ulam recognised that computers would permit the simulation of intractably complex systems.

> The first thoughts and attempts I made to practice [the Monte Carlo Method] were suggested by a question which occurred to me in 1946 as I was convalescing from an illness and playing solitaire. The question was what are the chances that a Canfield solitaire laid out with 52 cards will come out successfully? After spending a lot of time trying to estimate them by combinatorial calculations, I wondered whether a more practical method than "abstract thinking" might not be to lay it out say one hundred times and simply observe and count the number of successful plays. This was already possible to envisage with the beginning of the era of fast computers, and I immediately thought of problems of neutron diffusion and other questions of mathematical physics, and more generally how to change processes described by certain differential equations into an equivalent form interpretable as a succession of random operations. Later … [in 1946, I] described the idea to John von Neumann, and we began to plan actual calculations.
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> *Eckhardt, Roger (1987). Stan Ulam, John von Neumann and the Monte Carlo method, Los Alamos Science, Special Issue (15), 131-137*

METHOD DESCRIBED

By the Monte Carlo method, uncertain model parameters are randomly sampled from an appropriate probability distribution. The system is then run through time and the outcome computed. This realises many independent results, each of which is a possible "future". These realisations are then assembled into a probability distribution.

SIMPLE EXAMPLE

As a simple example, consider the probability of a given sum resulting from a throw of two dice. There are thirty-six (6^2) possible combinations of that system, six of which sum to the number seven. Hence, the probability of rolling a 7 with two dice is 6/36 or 16.66%.

However, instead of computing the "pure" probability in this way we could instead repeatedly throw the dice and record the outcomes, knowing that as the number of throws increases our results will converge to the abstract probability. Better yet, we could program a computer to simulate throwing the dice. This would permit an arbitrarily large number of "throws" and so give correspondingly greater confidence in the result.

ACCURACY

Assuming the probability distributions for the uncertain parameters are appropriately defined to begin with, the accuracy of the Monte Carlo method is a function of the number of realisations. Correspondingly, confidence bounds on results can be readily computed.

USAGE

Monte Carlo simulation has been engaged to address a diverse range of intractable problems. As has been observed:

> The [Monte Carlo method] continues to be one of the most useful approaches to scientific computing due to its simplicity and general applicability. The next generation of Monte Carlo techniques will provide important tools for solving ever more complex estimation and optimisation problems in engineering, finance, statistics, mathematics, computer science, and the physical and life sciences.
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> *Kroese, Dirk et al (2014). Why the Monte Carlo Method is so important today, Wiley Interdisciplinary Reviews: Computational Statistics 6, 386-392.*